Tangent Planes

1. Definition:

- Let $f: \mathbb{R}^2 \to \mathbb{R}$ be diff at (x_0, y_0). Then, the tangent plane of the graph of f at the point $(x_0, y_0, f(x_0, y_0))$ is given by

$$z = f(x_0, y_0) + \left[\frac{\partial f}{\partial x}(x_0, y_0)\right](x - x_0) + \left[\frac{\partial f}{\partial y}(x_0, y_0)\right](y - y_0)$$

2. Example:

Find the eqn of the tangent plane to $z = f(x,y) = \frac{y}{1+x^2+y^2}$ at the point $(2, 3, f(2,3))$.

Soln:

$$f(2,3) = \frac{3}{1+4+9} = \frac{3}{14}$$

$$\frac{\partial f}{\partial x} = \frac{0 - (y)(2x)}{(1+x^2+y^2)^2}$$

$$\frac{\partial f}{\partial x}(2,3) = \frac{-(3)(4)}{(1+4+9)^2} = \frac{-12}{196} = -\frac{3}{49}$$

$$\frac{\partial f}{\partial y} = \frac{(1+x^2+y^2) - (y)(2y)}{(1+x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y}(2,3) = \frac{(1+4+9) - (3)(6)}{196} = \frac{-4}{196} = \frac{-1}{49}$$

$$z = f(x_0, y_0) + \left[\frac{\partial f}{\partial x}(x_0, y_0)\right](x - x_0) + \left[\frac{\partial f}{\partial y}(x_0, y_0)\right](y - y_0)$$

$$z = \frac{3}{14} - \frac{3}{49}(x-2) - \frac{1}{49}(y-3)$$

$$98z = 21 - 6(x-2) - 2(y-3)$$

$$98z = 21 - (6x - 12) - (2y - 6)$$
$$= 21 - 6x + 12 - 2y + 6$$
$$-39 = -6x - 2y - 98z$$
$$39 = 6x + 2y + 98z \leftarrow \text{Eqn of tangent plane}$$